ROZBOR ZKOUŠKOVÝCH PŘÍKLADŮ

1) $\boxed{y = \arccos \frac{x-1}{2} \qquad \text{rovnoběžná s } y = -\frac{1}{2}x + 10} \longrightarrow y' = -\frac{1}{2}$

$$y' = \frac{-1}{\sqrt{1-\left(\frac{x-1}{2}\right)^2}} \cdot \frac{(1-0)\cdot 2 - \cancel{(x-1)\cdot 0}}{2^2} = \frac{-1}{\sqrt{1-\frac{x^2-2x+1}{4}}} \cdot \frac{\cancel{2}^{1}}{\cancel{4}_{2}} =$$

$$= \frac{-1}{\sqrt{\frac{4-x^2+2x-1}{4}}} \cdot \frac{1}{2} = \frac{-1}{\frac{\sqrt{3-x^2+2x}}{\cancel{2}}} \cdot \frac{1}{\cancel{2}} = \boxed{\frac{-1}{\sqrt{-x^2+2x+3}}}$$

bod dotyku $\quad \boxed{\frac{\cancel{-}1}{\sqrt{-x^2+2x+3}} = \cancel{-}\frac{1}{2}}$

$$2 = \sqrt{-x^2+2x+3} \quad /^2$$

$$4 = -x^2+2x+3$$

$$x^2-2x+1 = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{(-2)^2-4\cdot 1\cdot 1}}{2\cdot 1} = \frac{2 \pm \cancel{\sqrt{4-4}}}{2} = \frac{2}{2} = 1$$

$\quad\ x_0 \ \ y_0$

$T[1; ?]$

$$y_0 = y(x_0) = \arccos \frac{1-1}{2} = \arccos 0 = \frac{\pi}{2}$$

$$y'(x_0) = \frac{-1}{\sqrt{-1^2+2\cdot 1+3}} = \frac{-1}{\sqrt{-1+2+3}} = \frac{-1}{\sqrt{4}} = \frac{-1}{2}$$

$$\boxed{y_T = y(x_0) + y'(x_0)\cdot(x-x_0)}$$

$$y_T = \frac{\pi}{2} + \left(-\frac{1}{2}\right)\cdot(x-1) = \frac{\pi}{2} - \frac{x}{2} + \frac{1}{2} = -\frac{x}{2} + \frac{\pi}{2} + \frac{1}{2}$$

② $\boxed{y=\frac{1}{1+e^{-x}}}$

1) $D_f=\mathbb{R}$

$1+e^{-x}\neq 0$

$e^{-x}\neq -1$

vždy

e na cokoliv je vždy kladné

2) průsečíky

P_x $y=0$ $0=\frac{1}{1+e^{-x}}$

$0\neq 1$ nelze

průsečík s x neexistuje

P_y $x=0$ $y=\frac{1}{1+e^{-0}}$

$y=\frac{1}{1+1}$

$y=\frac{1}{2}$ $[0;\frac{1}{2}]$

$+$

$-\infty$ (0) ∞

$(0):\ \frac{1}{2}=+$

3) monotónnost

$$y'=\frac{\cancel{0\cdot(1+e^{-x})}-1\cdot(0+e^{-x}\cdot(-1))}{(1+e^{-x})^2}=\frac{+(+e^{-x})}{(1+e^{-x})^2}=\frac{e^{-x}}{(1+e^{-x})^2}$$

$y'=0 \longrightarrow \frac{e^{-x}}{(1+e^{-x})^2}=0$

$e^{-x}\neq 0$

$\nearrow$

$-\infty$ (0) ∞

$(0):\ \frac{e^{-0}}{(1+e^{-0})^2}=\frac{1}{(1+1)^2}=\frac{1}{4}=+$

4) konvexnost, konkávnost

$$y''=\frac{e^{-x}(-1)\cdot(1+e^{-x})^2-e^{-x}\cdot 2(1+e^{-x})^1\cdot(\cancel{0}+e^{-x}\cdot(-1))}{(1+e^{-x})^4}=\frac{-e^{-x}(1+e^{-x})^2+2e^{-x}e^{-x}(1+e^{-x})}{(1+e^{-x})^4}=$$

$$=\frac{e^{-x}\cancel{(1+e^{-x})}[-(1+e^{-x})+2e^{-x}]}{(1+e^{-x})^{\cancel{4}3}}=\frac{e^{-x}[-1-e^{-x}+2e^{-x}]}{(1+e^{-x})^3}=\frac{e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}$$

$y''=0 \longrightarrow \frac{e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}=0$

$e^{-x}\cdot(e^{-x}-1)=0$

$e^{-x}\neq 0$

$e^{-x}-1=0$

$e^{-x}=1$

$\cancel{\ln e}^{-x}=\ln 1$

$-x=0$

$x=0$

$\cup$ $\cap$

$-\infty$ (-1) 0 (1) ∞

2,7 − 1

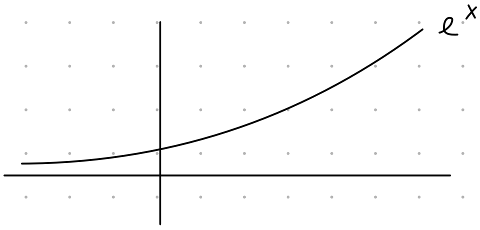
$(-1):\ \frac{e^{1}(e^{1}-1)}{(1+e^{1})^3}=\frac{+\cdot+}{(+)^3}=\frac{+}{+}=+$

1 + 2,7 1 − 2,7

$(1):\ \frac{e^{-1}(e^{-1}-1)}{(1+e^{-1})^3}=\frac{+\cdot(\frac{1}{e}-1)}{(+)^3}=\frac{+\cdot\frac{1-e}{e}}{+}=\frac{+\cdot\frac{-}{+}}{+}=-$

IB $x=0$ $y=\frac{1}{2}$ $[0;\frac{1}{2}]$

5) ABS $\quad D_f = \mathbb{R} = (-\infty; \infty)$ $\quad$ neexistují



$e^{\infty} = \infty$
$e^{-\infty} = 0$

6) ASS $\quad D_f = \mathbb{R} = (-\infty; \infty)$

pro $x \to -\infty$ $\qquad y = kx + q$

$$k_1: \lim_{x \to -\infty} \frac{\frac{1}{1+e^{-x}}}{x} = \lim_{x \to -\infty} \frac{1}{x(1+e^{-x})} = \frac{1}{-\infty(1+e^{\infty})} = \frac{1}{-\infty \cdot \infty} = \frac{1}{-\infty} = 0$$

$$q_1: \lim_{x \to -\infty} \frac{1}{1+e^{-x}} - \cancel{0 \cdot x} = \frac{1}{1+e^{\infty}} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$$

$y_1 = 0x + 0 = 0 \quad$ pro $x \to -\infty$

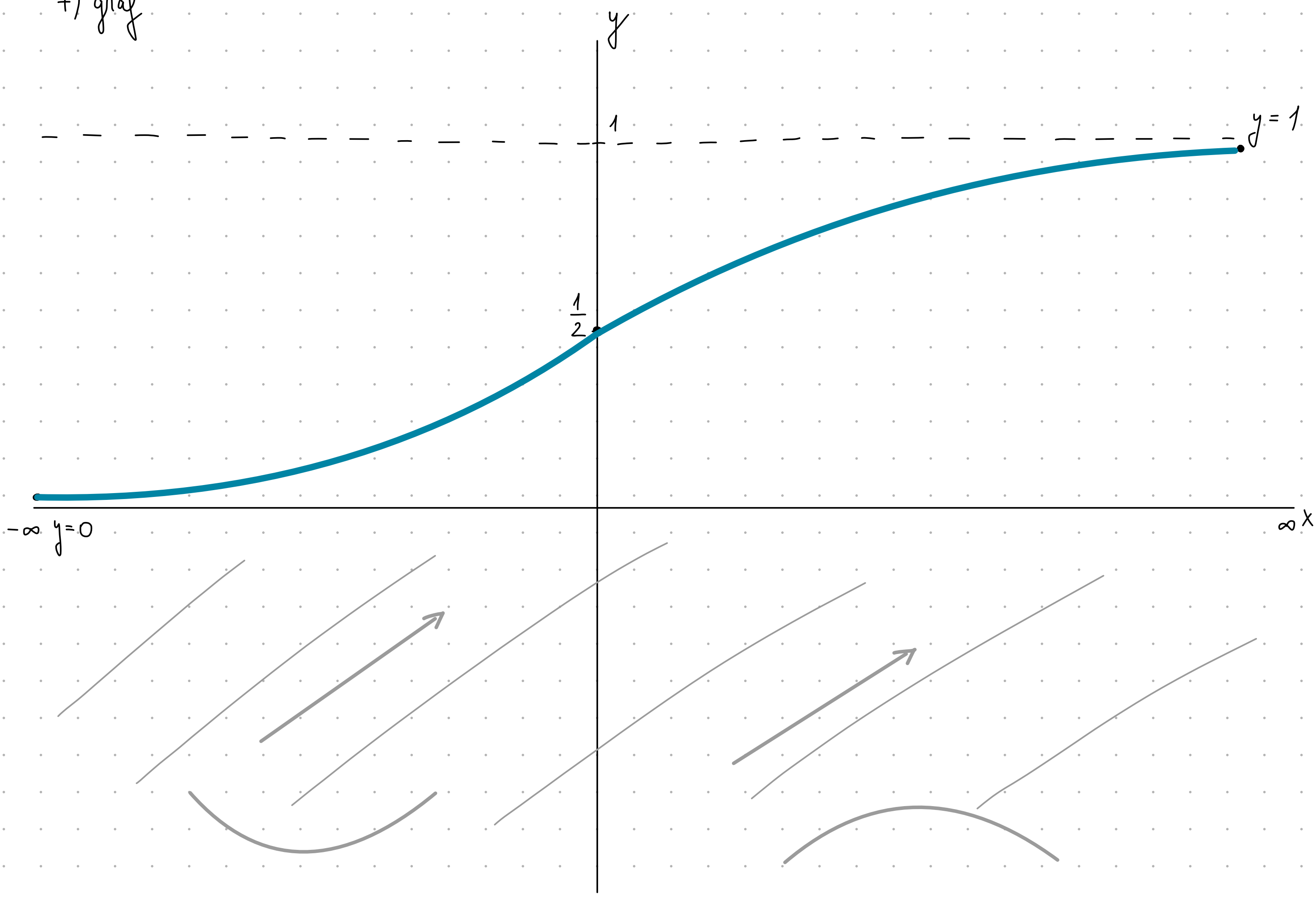
pro $x \to \infty$ $\qquad y = kx + q$

$$k_2: \lim_{x \to \infty} \frac{1}{x(1+e^{-x})} = \frac{1}{\infty \cdot (1+e^{-\infty})} = \frac{1}{\infty(1+0)} = \frac{1}{\infty} = 0$$

$$q_2: \lim_{x \to \infty} \frac{1}{1+e^{-x}} - \cancel{0 \cdot x} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0} = 1$$

$y_2 = 0x + 1 = 1 \quad$ pro $x \to \infty$

7) graf



③ a)

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & -1 & -1 \\ 2 & -5 & 1 & -2 & -2 \\ -2 & -1 & 1 & -2 & -3 \\ 3 & -2 & -1 & 1 & 2 \end{array}\right) (-2)\ (2)\ (-3) \sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & -1 & -1 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & -5 & 3 & -4 & -5 \\ 0 & 4 & -4 & 4 & 5 \end{array}\right) (-5)\ (4) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & -1 & -1 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 8 & -4 & -5 \\ \cancel{0} & \cancel{0} & \cancel{-8} & \cancel{4} & \cancel{5} \end{array}\right) (-4) \sim \left(\begin{array}{cccc|c} -4 & 8 & 4 & 0 & -1 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 8 & -4 & -5 \end{array}\right) (8)$$

$$\sim \left(\begin{array}{cccc|c} -4 & 0 & -4 & 0 & -1 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 8 & -4 & -5 \end{array}\right) \begin{array}{l} :(-4) \\ :(-1) \\ :(-4) \end{array} \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & \frac{1}{4} \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & \frac{5}{4} \end{array}\right) \end{array} \begin{array}{l} \mathrm{I.} \\ \mathrm{II.} \\ \mathrm{III.} \end{array}$$

III. $-2x_3 + x_4 = \frac{5}{4}$
$x_4 = \frac{5}{4} + 2x_3$

II. $x_2 + x_3 = 0$
$x_2 = -x_3$

I. $x_1 + x_3 = \frac{1}{4}$
$x_1 = \frac{1}{4} - x_3$

OŘ: $\vec{x} = \left(\frac{1}{4} - x_3;\ -x_3;\ x_3;\ \frac{5}{4} + 2x_3\right)$

ZŘ: $\vec{x} = \left(\frac{1}{4};\ 0;\ 0;\ \frac{5}{4}\right)$

b)

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & \frac{1}{4} \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & \frac{5}{4} \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & -2 & 1 & \frac{5}{4} \end{array}\right) (2) \sim$$

$$\sim \left(\begin{array}{cccc|c} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & \frac{1}{4} \\ 2 & 0 & 0 & 1 & \frac{7}{4} \end{array}\right) (-1) \sim \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left(\begin{array}{cccc|c} -1 & 1 & 0 & 0 & -\frac{1}{4} \\ 1 & 0 & 1 & 0 & \frac{1}{4} \\ 2 & 0 & 0 & 1 & \frac{7}{4} \end{array}\right) \end{array} \begin{array}{l} \mathrm{I.} \\ \mathrm{II.} \\ \mathrm{III.} \end{array}$$

I. $-x_1 + x_2 = -\frac{1}{4}$
$x_2 = -\frac{1}{4} + x_1$

II. $x_1 + x_3 = \frac{1}{4}$
$x_3 = \frac{1}{4} - x_1$

III. $2x_1 + x_4 = \frac{7}{4}$
$x_4 = \frac{7}{4} - 2x_1$

OŘ: $\vec{x} = \left(x_1;\ -\frac{1}{4} + x_1;\ \frac{1}{4} - x_1;\ \frac{7}{4} - 2x_1\right)$

ZŘ: $\vec{x} = \left(0;\ -\frac{1}{4};\ \frac{1}{4};\ \frac{7}{4}\right)$

④ $\boxed{y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 20x + 6}$

$$y' = \frac{1}{\cancel{3}} \cdot \cancel{3}x^2 + \frac{1}{\cancel{2}} \cdot \cancel{2}x^1 - 20 = x^2 + x - 20$$

$$y' = 0 \longrightarrow x^2 + x - 20 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-20)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1+80}}{2} = \frac{-1 \pm 9}{2} \begin{cases} 4 \\ -5 \end{cases}$$

$$y'' = 2x + 1$$

$y''(4) = 2 \cdot 4 + 1 = 9 \quad > 0$ **L.MIN**

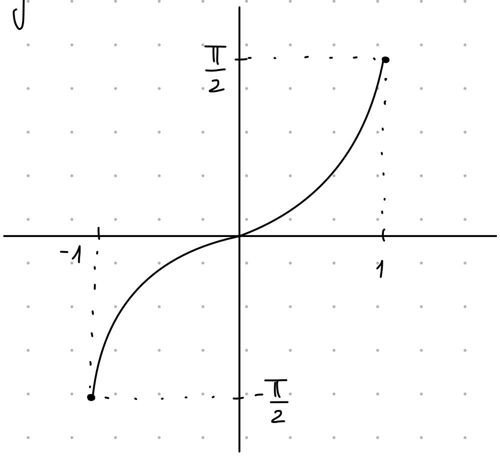
$y''(-5) = 2(-5) + 1 = -9 \quad < 0$ **L.MAX**

L MIN
$x = 4$
$y = \frac{1}{3} \cdot 4^3 + \frac{1}{2} 4^2 - 20 \cdot 4 + 6 = \frac{-134}{3}$ $\left[4; -\frac{134}{3}\right]$

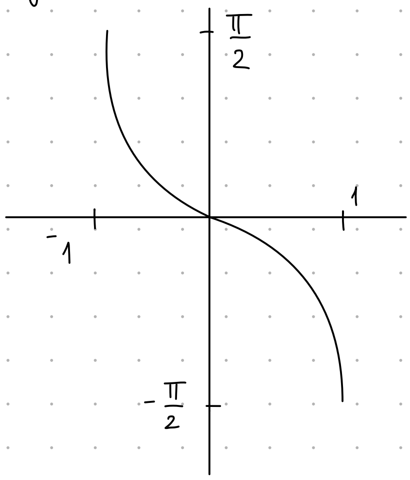
L.MAX
$x = -5$
$y = \frac{1}{3} \cdot (-5)^3 + \frac{1}{2} \cdot (-5)^2 - 20 \cdot (-5) + 6 = \frac{461}{6}$ $\left[-5; \frac{461}{6}\right]$

5 $\boxed{y = \frac{\pi}{2} - \arcsin(x-1)}$

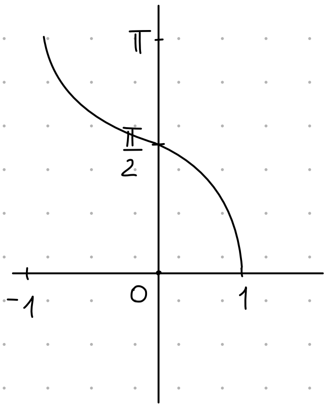
$y = \arcsin x$



$y = -\arcsin x$

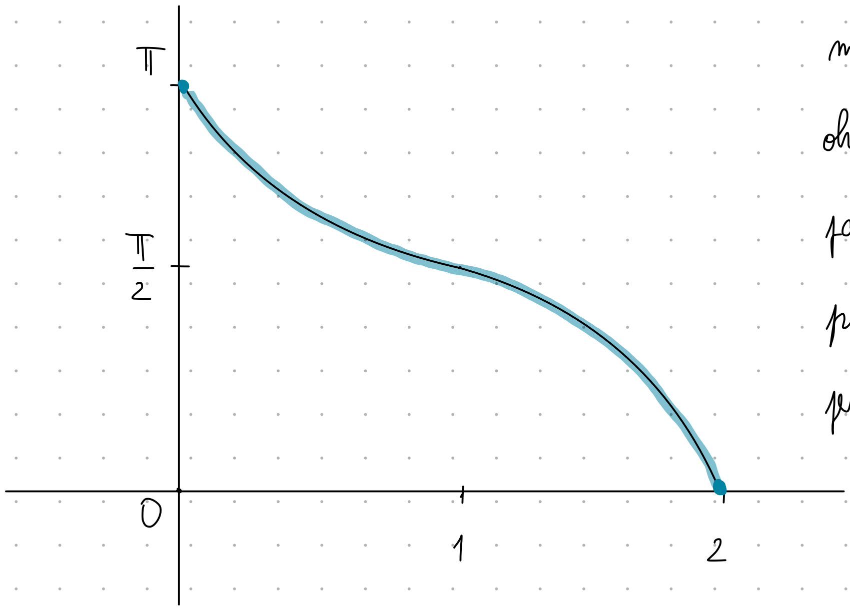


$y = \frac{\pi}{2} - \arcsin x$



$y = \frac{\pi}{2} - \arcsin(x-1)$

$x - 1 = 0$
$x = 1$



$D_f = \langle 0; 2 \rangle$

$H_f = \langle 0; \pi \rangle$

monotónnost = ryze monotónní

ohraničenost = ohraničená

parita = není ani sudá ani lichá

prostost = prostá

periodičnost = není periodické

(6) $d: \ Q = 8 - 0{,}5P \qquad TC(Q) = Q^2 + 9Q + 2$

- inverzní poptávka

$$0{,}5P = 8 - Q \quad | \cdot 2$$
$$P = 16 - 2Q$$

- celkový příjem

$$AR = 16 - 2Q$$
$$TR = AR \cdot Q = (16 - 2Q) \cdot Q = 16Q - 2Q^2$$

- zisk

$$PR = TR - TC = 16Q - 2Q^2 - (Q^2 + 9Q + 2) = -3Q^2 + 7Q - 2$$

- max. zisk

$$PR'_Q = \frac{\partial PR}{\partial Q} = -3 \cdot 2Q^1 + 7 = -6Q + 7$$

$$PR'_Q = 0 \longrightarrow -6Q + 7 = 0$$
$$7 = 6Q \quad | : 6$$
$$Q = \frac{7}{6}$$

$$PR\left(\frac{7}{6}\right) = -3 \cdot \left(\frac{7}{6}\right)^2 + 7 \cdot \frac{7}{6} - 2 = \frac{25}{12}$$

7) $$\boxed{z = \frac{x}{y} + \frac{2}{x} - 4y + 5}$$

1) D_f: $x \neq 0 \ \wedge \ y = 0$

2) $z'_x = 1 \cdot \frac{1}{y} + 2 \cdot (-1) \cdot x^{-2} = \frac{1}{y} - \frac{2}{x^2} = \frac{x^2 - 2y}{x^2 y}$

$z'_y = x \cdot (-1) y^{-2} - 4 = \frac{-x}{y^2} - 4 = \frac{-x - 4y^2}{y^2}$

3) $\frac{x^2 - 2y}{x^2 y} = 0$

$x^2 - 2y = 0$

$-2y = -x^2 \quad |:(-2)$

$y = \frac{x^2}{2}$

$\frac{-x - 4y^2}{y^2} = 0$

$-x - 4y^2 = 0$

$-x - 4 \cdot \left(\frac{x^2}{2}\right)^2 = 0$

$-x - \cancel{4} \cdot \frac{x^4}{\cancel{4}} = 0$

$-x\,(1 + x^3) = 0$

$-x = 0$

$\underline{x = 0}$

$1 + x^3 = 0$

$x^3 = -1$

$\underline{x = -1}$

$x = 0 \qquad y = \frac{0^2}{2} = 0 \qquad P_1\,[0;\,0] \notin D_f$

$x = -1 \qquad y = \frac{(-1)^2}{2} = \frac{1}{2} \qquad P_2\,[-1;\,\frac{1}{2}]$

4) $z''_{xx} = \frac{2x \cdot x^2 y - (x^2 - 2y) \cdot 2x \cdot y}{(x^2 y)^2} = \frac{2x\cancel{y}\,[\cancel{x^2} - \cancel{x^2} + 2y]}{x^{\cancel{4}\,3}\, y^{\cancel{2}}} = \frac{4\cancel{y}}{x^3 \cancel{y}} = \frac{4}{x^3}$

$z''_{yy} = \frac{-8y \cdot y^2 - (-x - 4y^2) \cdot 2y}{(y^2)^2} = \frac{2\cancel{y}\,[\cancel{-4y^2} + x + \cancel{4y^2}]}{y^{\cancel{4}\,3}} = \frac{2x}{y^3}$

$z''_{xy} = \frac{-2 \cdot x^2 y - (x^2 - 2y) \cdot x^2 \cdot 1}{(x^2 y)^2} = \frac{\cancel{x^2}\,[\cancel{-2y} - x^2 + \cancel{2y}]}{x^{\cancel{4}\,2} y^2} = \frac{-\cancel{x^2}}{\cancel{x^2} y^2} = \frac{-1}{y^2}$

5) $z''_{xx}(P_2) = \frac{4}{(-1)^3} = -4$

$z''_{yy}(P_2) = \frac{2 \cdot (-1)}{\left(\frac{1}{2}\right)^3} = \frac{-2}{\frac{1}{8}} = -16$

$z''_{xy}(P_2) = \frac{-1}{\left(\frac{1}{2}\right)^2} = \frac{-1}{\frac{1}{4}} = -4$

6) $-4 < 0$ L.MAX

$$D = \begin{vmatrix} -4 & -4 \\ -4 & -16 \end{vmatrix} = 4 \cdot 16 - 4 \cdot 4 = 4 \cdot (16 - 4) = + \cdot + = +$$

7) $z = \frac{-1}{\frac{1}{2}} + \frac{2}{-1} - 4 \cdot \frac{1}{2} + 5 = -2 - 2 - 2 + 5 = -1$

8) L.MAX -1 v bodě $[-1;\,\frac{1}{2}]$

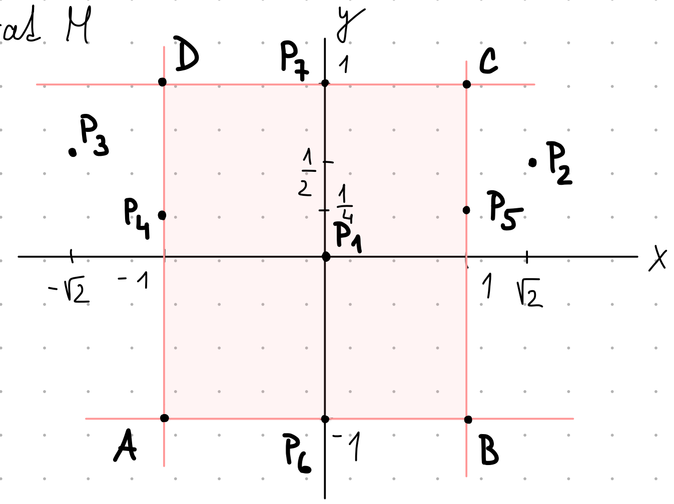
8) $z = x^2 + 4y^2 - 2x^2y + 4$

$M: -1 \le x \le 1 \longrightarrow -1 \le x \;;\; x \le 1$

$-1 \le y \le 1 \longrightarrow -1 \le y \;;\; y \le 1$

1) $D_f = \mathbb{R} \times \mathbb{R}$

2) namalovat M



$A\,[-1; -1]$

$B\,[1; -1]$

$C\,[1; 1]$

$D\,[-1; 1]$

3) lokální extrémy

$z'_x = 2x - 4xy$

$2x - 4xy = 0$

$2x - \cancel{4}x \cdot \frac{1}{\cancel{4}}x^2 = 0$

$2x - x^3 = 0$

$x \cdot (2 - x^2) = 0$

$x = 0$

$2 - x^2 = 0$

$2 = x^2$

$\pm\sqrt{2} = x$

$z'_y = 8y - 2x^2$

$8y - 2x^2 = 0$

$8y = 2x^2 \quad | : 8$

$y = \frac{2}{8}x^2$

$y = \frac{1}{4}x^2$

$\sqrt{2} \doteq 1{,}4$

- $x = 0 \qquad y = \frac{1}{4} \cdot 0^2 = 0 \qquad P_1\,[0; 0] \in M$
- $x = \sqrt{2} \qquad y = \frac{1}{4}(\sqrt{2})^2 = \frac{1}{4} \cdot 2 = \frac{1}{2} \qquad P_2\,[\sqrt{2}; \frac{1}{2}] \notin M$
- $x = -\sqrt{2} \qquad y = \frac{1}{4} \cdot (-\sqrt{2})^2 = \frac{1}{4} \cdot 2 = \frac{1}{2} \qquad P_3\,[-\sqrt{2}; \frac{1}{2}] \notin M$

4) vázané extrémy

a) $x = -1$

$z = (-1)^2 + 4y^2 - 2(-1)^2y + 4$

$z = 1 + 4y^2 - 2y + 4$

$z'_y = 8y - 2$

$8y - 2 = 0$

$8y = 2$

$y = \frac{1}{4}$

$P_4\,[-1; \frac{1}{4}] \in M$

b) $x = 1$

$z = 1^2 + 4y^2 - 2 \cdot 1^2 y + 4$

$z = 1 + 4y^2 - 2y + 4$

$z'_y = 8y - 2$

$8y - 2 = 0$

$8y = 2$

$y = \frac{1}{4}$

$P_5\,[1; \frac{1}{4}] \in M$

c) $y=-1$

$z = x^2 + 4(-1)^2 - 2x^2\cdot(-1) + 4$

$z = x^2 + 4 + 2x^2 + 4$

$z'_x = 2x + 4x$ $\qquad$ $2x+4x=0$

$6x=0$

$P_6\,[0;-1] \in M$ $\qquad$ $x=0$

d) $y=1$

$z = x^2 + 4\cdot 1^2 - 2x^2\cdot 1 + 4$

$z = x^2 + 4 - 2x^2 + 4$

$z'_x = 2x - 4x$ $\qquad$ $2x-4x=0$

$-2x=0$

$P_7\,[0;1] \in M$ $\qquad$ $x=0$

5) vyhodnocení

	$z = x^2 + 4y^2 - 2x^2y + 4$	
$[-1;-1]$ A	$(-1)^2 + 4(-1)^2 - 2\cdot(-1)^2\cdot(-1) + 4 = 1+4+2+4 = 11$	ABS. MAX
$[1;-1]$ B	$1^2 + 4\cdot(-1)^2 - 2\cdot 1^2\cdot(-1) + 4 = 1+4+2+4 = 11$	ABS. MAX
$[1;1]$ C	$1^2 + 4\cdot 1^2 - 2\,1^2\cdot 1 + 4 = 1+4-2+4 = 7$	
$[-1;1]$ D	$(-1)^2 + 4\cdot 1^2 - 2\,(-1)^2\cdot 1 + 4 = 1+4-2+4 = 7$	
$[0;0]$ P_1	$\cancel{0^2} + \cancel{4\,0^2} - \cancel{2\cdot 0^2}\cdot\cancel{0} + 4 = 4$	ABS. MIN
$[-1;\frac{1}{4}]$ P_4	$(-1)^2 + 4\cdot\left(\frac{1}{4}\right)^2 - 2\cdot(-1)^2\cdot\frac{1}{4} + 4 = 1 + 4\cdot\frac{1}{16} - 2\cdot\frac{1}{4} + 4 = 5 + \frac{1}{4} - \frac{2}{4} = 5 - \frac{1}{4} = \frac{20-1}{4} = \frac{19}{4} \doteq 4{,}75$	
$[1;\frac{1}{4}]$ P_5	$1^2 + 4\left(\frac{1}{4}\right)^2 - 2\cdot 1^2\cdot\frac{1}{4} + 4 = \frac{19}{4} \doteq 4{,}75$	
$[0;-1]$ P_6	$\cancel{0^2} + 4\,(-1)^2 - \cancel{2\cdot 0^2(-1)} + 4 = 4+4 = 8$	
$[0;1]$ P_7	$\cancel{0^2} + 4\cdot 1^2 - \cancel{2\cdot 0^2\cdot 1} + 4 = 4+4 = 8$	

⑨ $\boxed{y' + \frac{y}{x-1} = e^{-x}}$

$$\left.\begin{aligned} a(x) &= \frac{1}{x-1} \\ b(x) &= e^{-x} \end{aligned}\right\} \text{ LIN.}$$

$$y = e^{-\int \frac{1}{x-1} dx} \cdot \left(C + \int e^{-x} \cdot e^{\int \frac{1}{x-1} dx} dx \right)$$

$$\longrightarrow \int \underbrace{\frac{1}{x-1}}_{1} dx = \ln|x-1|$$

$$y = e^{-\ln|x-1|} \cdot \left(C + \int e^{-x} \cdot \cancel{e}^{\ln|x-1|} dx \right)$$

$$y = \cancel{e}^{\ln|x-1|^{-1}} \cdot \left(C + \int e^{-x} \cdot (x-1)\, dx \right)$$

$$\longrightarrow \int e^{-x} \cdot (x-1) dx = \left| \begin{matrix} u = x-1 & u' = 1 \\ v' = e^{-x} & v = e^{-x} \cdot \frac{1}{-1} = -e^{-x} \end{matrix} \right| =$$

$$= (x-1) \cdot (-e^{-x}) - \int 1 \cdot (-e^{-x}) dx = -(x-1) \cdot e^{-x} + \int e^{-x} dx =$$

$$= -(x-1) \cdot e^{-x} + e^{-x} \cdot \frac{1}{-1} = -e^{-x} [x \cancel{-1} \cancel{+1}] = -x \cdot e^{-x}$$

$$y = \frac{1}{x-1} \left(C - x e^{-x} \right) = \boxed{\frac{C - x e^{-x}}{x-1}}$$ obecné řešení

P.P. $y(-1) = 0$

$$0 = \frac{C + 1 \cdot e^{1}}{-1-1}$$

$$0 = C + e$$

$$C = -e \longrightarrow \boxed{y_P = \frac{-e - x e^{-x}}{x-1}}$$

(10)

$$\boxed{z = x^2 \cdot \ln \frac{4-x^2-y^2}{x+4y} + \operatorname{arctg}\,(x - \ln x)}$$

$\frac{4-x^2-y^2}{x+4y} > 0$ $\qquad x+4y \neq 0$ ✓ $\qquad x > 0$

$\frac{+}{+}$ $\qquad \frac{-}{-}$

$4-x^2-y^2 > 0 \;\wedge\; x+4y > 0 \qquad\qquad 4-x^2-y^2 < 0 \;\wedge\; x+4y < 0$

$4 > x^2+y^2$ $\qquad 4y > -x$ $\qquad 4 < x^2+y^2$ $\qquad 4y < -x$

$y > -\frac{x}{4}$ $\qquad y < -\frac{x}{4}$

$S[0;0]\,;\, r=2$ $\qquad S[0;0]\,;\, r=2$

dosadit $[0;0]$ UVNITŘ

$4 > 0^2+0^2$

$4 > 0$ ANO

$y=0 \quad 0 = -\frac{x}{4}$

$0 = x \quad [0;0]$

$y=1 \quad 1 = -\frac{x}{4}$

$4 = -x$

$-4 = x \quad [-4;1]$

dosadit $[2;2]$ NAHOŘE

$2 > -\frac{2}{4}$

$2 > -\frac{1}{2}$ ANO

dosadit $[0;0]$ UVNITŘ

$4 < 0^2 + 0^2$

$4 < 0$ **NE**

$y=0 \quad 0 = -\frac{x}{4}$

$0 = x \quad [0;0]$

$y=1 \quad 1 = -\frac{x}{4}$

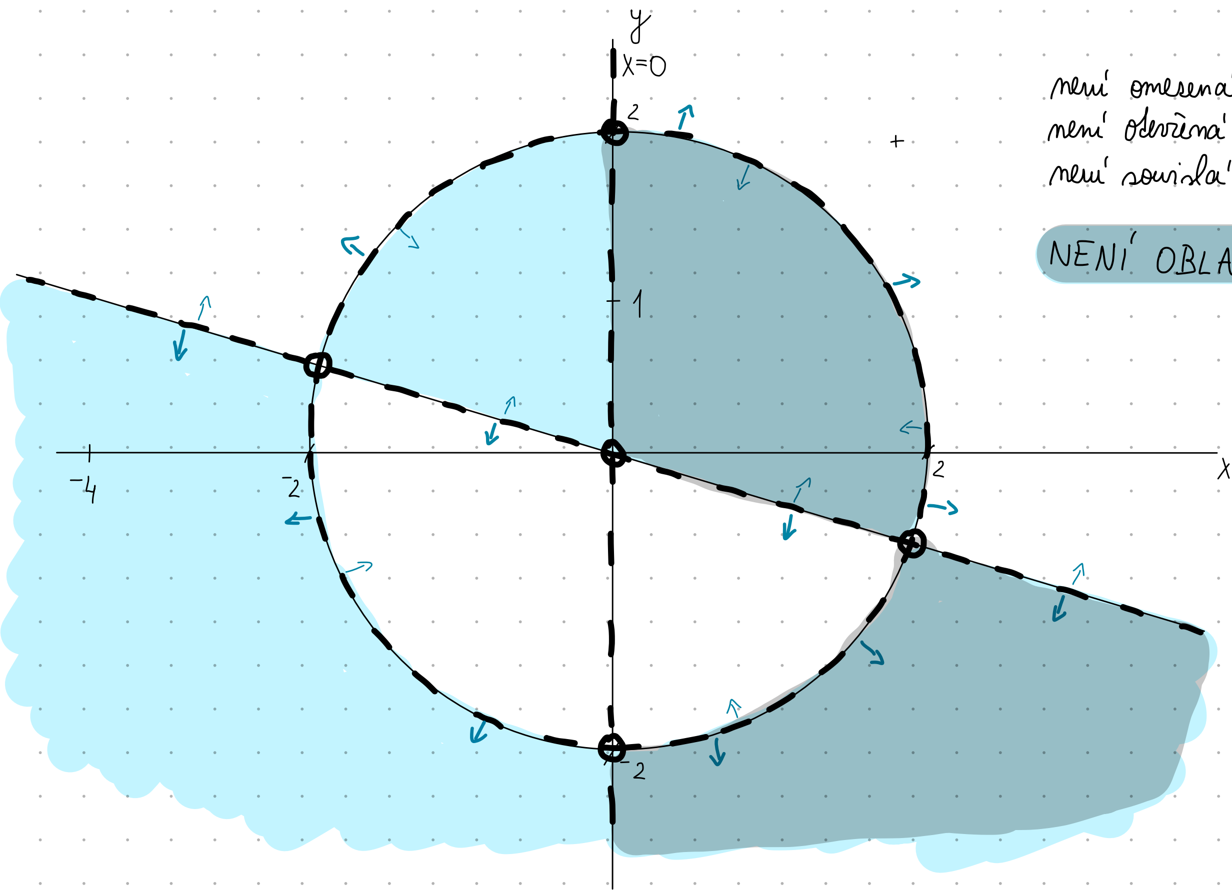
$4 = -x$

$-4 = x \quad [-4;1]$

dosadit $[2;2]$ NAHOŘE

$2 < -\frac{2}{4}$

$2 < -\frac{1}{2}$ **NE**



není omezená

není otevřená ani uzavřená

není souvislá

NENÍ OBLAST

11 $\quad 10 \to 5M, 2Z, 3Č$ $\quad P(M) = \frac{5}{10}$ $\quad P(Z) = \frac{2}{10}$ $\quad P(Č) = \frac{3}{10}$

POSTUPNĚ

a) 1. M ; 2. Z ; 3. Č *vracíme*

$$P(A) = \frac{\cancel{5}}{\cancel{10}_{2}} \cdot \frac{\cancel{2}}{10} \cdot \frac{3}{10} = \boxed{\frac{3}{100}}$$

b) 1. M ; 2 Č ; 3. M *nevracíme*

$$P(B) = \frac{\cancel{5}}{\cancel{10}_{2}} \cdot \frac{\cancel{3}}{\cancel{9}_{3}} \cdot \frac{\cancel{4}}{\cancel{8}_{2}} = \boxed{\frac{1}{12}}$$