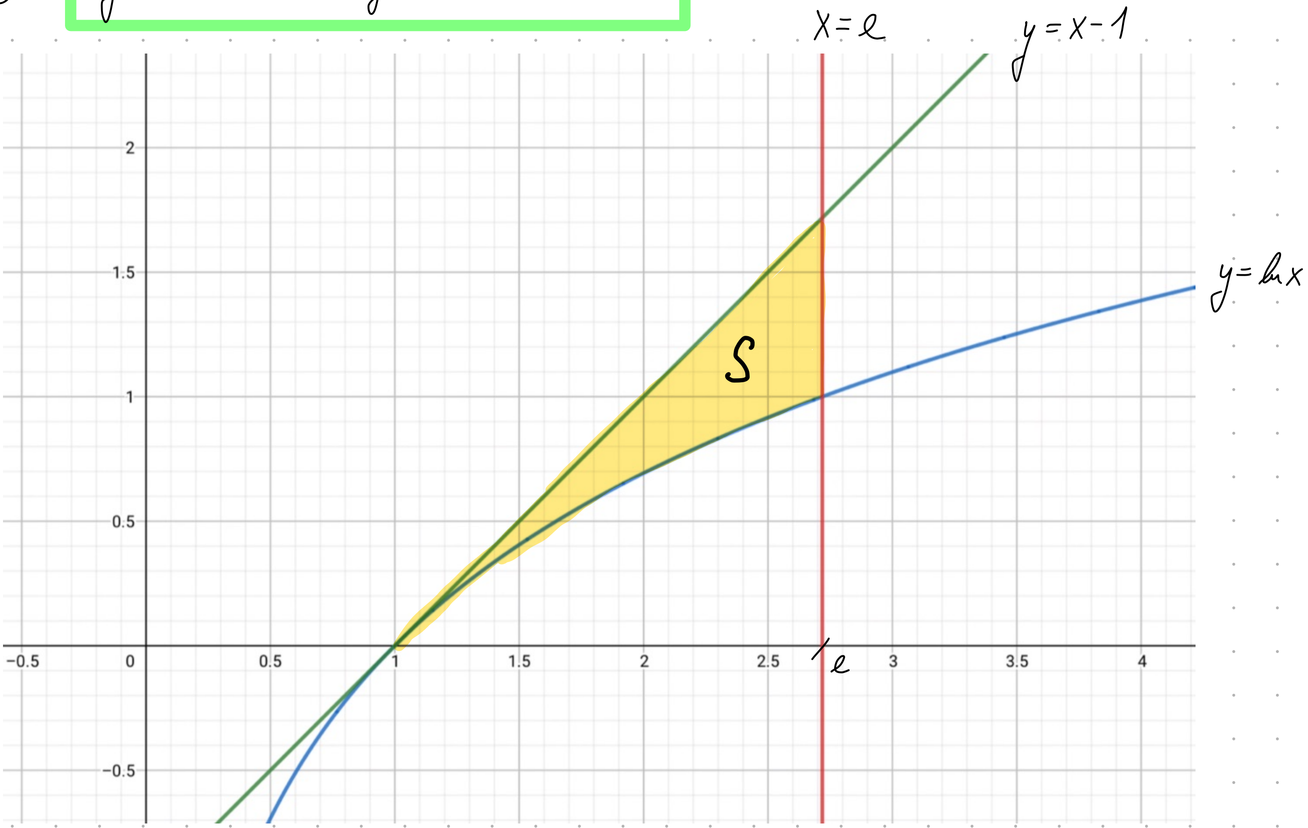
ROZBOR ZKOUŠKOVÝCH PŘÍKLADŮ 2

1. $y = x - 1 \qquad y = \ln x \qquad x = e$



$$S = \int_1^e x - 1 - \ln x \, dx = \int_1^e x - 1 \, dx - \int_1^e \ln x \, dx =$$

$$\int 1 \cdot \ln x \, dx = \left| \begin{matrix} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{matrix} \right| = x \cdot \ln x - \int \frac{1}{\cancel{x}} \cdot \cancel{x} \, dx = x \cdot \ln x - x = x(\ln x - 1)$$

$$= \left[\frac{x^2}{2} - x - x \cdot (\ln x - 1) \right]_1^e = \frac{e^2}{2} - e - e(\underbrace{\ln e}_{1} - 1) - \left[\frac{1}{2} - 1 - 1 \cdot (\underbrace{\ln 1}_{0} - 1) \right] =$$

$$= \frac{e^2}{2} - e - e \cdot (\underbrace{1 - 1}_{0}) - \frac{1}{2} + 1 + 1 \cdot (0 - 1) = \frac{e^2}{2} - e - 0 - \frac{1}{2} + \cancel{1} - \cancel{1} =$$

$$= \frac{e^2}{2} - e - \frac{1}{2} = \boxed{\frac{e^2 - 2e - 1}{2}}$$

② $z = 2x + 2y$ $\qquad g: x^2 + y^2 - 1 = 0$ $\quad |(-1)$

$1 - x^2 - y^2 = 0$

1) $\mathcal{D}f = \mathbb{R} \times \mathbb{R}$

$\lambda - \lambda x^2 - \lambda \cdot y^2$

2) $\boxed{L = 2x + 2y + \lambda \cdot (1 - x^2 - y^2)}$

3) $L'_x = 2 - 2x \cdot \lambda$

$L'_y = 2 - 2y \cdot \lambda$

$L'_\lambda = g = 1 - x^2 - y^2$

4)

$2 - 2x \cdot \lambda = 0$

$-2x\,\lambda = -2 \quad |(-1)$

$2x\,\lambda = 2$

$\lambda = \frac{\cancel{2}\,1}{\cancel{2}x}$

$\lambda = \frac{1}{x}$

$2 - 2y \cdot \lambda = 0$

$2y\,\lambda = 2$

$\lambda = \frac{\cancel{2}\,1}{\cancel{2}y}$

$\lambda = \frac{1}{y}$

$\frac{1}{x} = \frac{1}{y}$

$y = x$

$1 - x^2 - y^2 = 0$

$1 - x^2 - x^2 = 0$

$1 - 2x^2 = 0$

$1 = 2x^2$

$\frac{1}{2} = x^2$

$\pm\sqrt{\frac{1}{2}} = x$

$x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{1}{\sqrt{2}} \qquad y = \frac{1}{\sqrt{2}} \qquad \lambda = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} \qquad P_1\left[\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right] \quad \lambda = \sqrt{2}$

$x = -\frac{1}{\sqrt{2}} \qquad y = -\frac{1}{\sqrt{2}} \qquad \lambda = \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2} \qquad P_2\left[-\frac{1}{\sqrt{2}}; -\frac{1}{\sqrt{2}}\right] \quad \lambda = -\sqrt{2}$

5)	6) $P_1\left[\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right]$ $\lambda = \sqrt{2}$	$P_2\left[-\frac{1}{\sqrt{2}}; -\frac{1}{\sqrt{2}}\right]$ $\lambda = -\sqrt{2}$
$L''_{xx} = -2\lambda$	$-2\sqrt{2}$	$-2 \cdot (-\sqrt{2}) = 2\sqrt{2}$
$L''_{yy} = -2\lambda$	$-2\sqrt{2}$	$2\sqrt{2}$
$L''_{xy} = 0$	0	0
$g'_x = -2x$	$-2 \cdot \frac{1}{\sqrt{2}} = \frac{-2}{\sqrt{2}} = \frac{-\sqrt{2}\sqrt{2}}{\cancel{\sqrt{2}}} = -\sqrt{2}$	$-2\left(-\frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$
$g'_y = -2y$	$-\sqrt{2}$	$\sqrt{2}$

7)

$$D_{P_1} = - \begin{vmatrix} -2\sqrt{2} & 0 & -\sqrt{2} \\ 0 & -2\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & -\sqrt{2} & 0 \end{vmatrix} = -[0-(-8\sqrt{2})] = -[8\sqrt{2}] = -8\sqrt{2}$$

$$\begin{matrix} -2\sqrt{2} & 0 & -\sqrt{2} \\ 0 & -2\sqrt{2} & -\sqrt{2} \end{matrix}$$

$-4\sqrt{2}$
$-4\sqrt{2}$
0
$-8\sqrt{2}$

0
0
0
0

$-$

LOK. VÁZ. MAX.

$$D_{P_2} = - \begin{vmatrix} 2\sqrt{2} & 0 & \sqrt{2} \\ 0 & 2\sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 0 \end{vmatrix} = -[0-8\sqrt{2}] = 8\sqrt{2} = +$$

$$\begin{matrix} 2\sqrt{2} & 0 & \sqrt{2} \\ 0 & 2\sqrt{2} & \sqrt{2} \end{matrix}$$

$4\sqrt{2}$
$4\sqrt{2}$
0
$8\sqrt{2}$

0
0
0
0

LOK. VÁZ. MIN

8) L. VÁZ. MAX $P_1\left[\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\right]$ $\lambda = \sqrt{2}$

v bodě $z = 2 \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

L. VÁZ. MIN $P_2\left[-\frac{1}{\sqrt{2}}; -\frac{1}{\sqrt{2}}\right]$ $\lambda = -\sqrt{2}$

v bodě $z = 2 \cdot \left(-\frac{1}{\sqrt{2}}\right) + 2 \cdot \left(-\frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}} = -\sqrt{2} - \sqrt{2} = -2\sqrt{2}$

③ 3 kostky | kostky různé

a) větší než 5 $\longrightarrow$ $6, 7, 8, \dots, 18$

DOPLNĚK $\longrightarrow$ $\cancel{0}, \cancel{1}, \cancel{2}, 3, 4, 5$

$3: \quad 1+1+1 \quad \longrightarrow 1$

$4: \quad 2+1+1;\ 1+2+1;\ 1+1+2 \quad \longrightarrow 3$

$5: \quad 2+2+1;\ 2+1+2;\ 1+2+2;\ 3+1+1;\ 1+3+1;\ 1+1+3 \quad \longrightarrow 6$

$m(\bar{A}) = 1+3+6 = 10$

$m(\Omega) = 6 \cdot 6 \cdot 6 = 216$

$P(\bar{A}) = \frac{10}{216}$

$$P(A) = 1 - P(\bar{A}) = 1 - \frac{10}{216} = \frac{216-10}{216} = \boxed{\frac{206}{216}}$$

b) menší než 17 $\longrightarrow$ $3, 4, 5, \dots, 16$

DOPLNĚK $\longrightarrow$ $17, 18$

$17: \quad 5+6+6;\ 6+5+6;\ 6+6+5 \quad \longrightarrow 3$

$18: \quad 6+6+6 \quad \longrightarrow 1$

$m(\bar{B}) = 3+1 = 4$

$m(\Omega) = 6 \cdot 6 \cdot 6 = 216$

$$P(B) = 1 - P(\bar{B}) = 1 - \frac{4}{216} = \frac{216-4}{216} = \boxed{\frac{212}{216}}$$

④ $P(A)$ = 1. jakost $\quad$ $P(H_1)$ = 1. stroj

$P(\bar{A})$ = 2. jakost $\quad$ $P(H_2)$ = 2. stroj

$P(H_3)$ = 3 stroj

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celkem $200 + 100 + 300 = 600$

$$P(H_1) = \frac{200}{600} = \frac{2}{6} \qquad P(A|H_1) = 0{,}96$$

$$P(H_2) = \frac{100}{600} = \frac{1}{6} \qquad P(A|H_2) = 0{,}96$$

$$P(H_3) = \frac{300}{600} = \frac{3}{6} \qquad P(A|H_3) = 0{,}9$$

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a) $P(A) = P(A|H_1) \cdot P(H_1) + P(A|H_2) \cdot P(H_2) + P(A|H_3) \cdot P(H_3)$

$$P(A) = 0{,}96 \cdot \frac{2}{6} + 0{,}96 \cdot \frac{1}{6} + 0{,}9 \cdot \frac{3}{6} = 0{,}96 \cdot \left(\frac{2}{6} + \frac{1}{6}\right) + 0{,}9 \cdot \frac{3}{6} =$$

$$= 0{,}96 \cdot \frac{3}{6} + 0{,}9 \cdot \frac{3}{6} = \frac{3}{6} \cdot (0{,}96 + 0{,}9) = \frac{3}{6} \cdot 1{,}86 = \frac{1}{2} \cdot 1{,}86 = 0{,}93$$

b) 100 % víme, že je první jakosti

$$P(H_2|A) = \frac{P(H_2 \cap A)}{P(A)} = \frac{P(A|H_2) \cdot P(H_2)}{0{,}93} = \frac{0{,}96 \cdot \frac{1}{6}}{0{,}93} = \frac{0{,}16}{0{,}93} = \frac{16}{93}$$

⑤

$$\boxed{\begin{aligned} Y &= C + I + G \\ C &= a + bY \end{aligned}}$$

Jordanova metoda

neznáme nahoru

$$\begin{aligned} Y - C &= I + G \\ -bY + C &= a \end{aligned} \longrightarrow \begin{array}{c} \begin{array}{cc} Y & C \end{array} \\ \left(\begin{array}{cc|c} 1 & -1 & I+G \\ -b & 1 & a \end{array}\right) \end{array} b \sim$$

$$\sim \left(\begin{array}{cc|c} 1 & -1 & I+G \\ 0 & -b+1 & b(I+G)+a \end{array}\right) (-b+1)$$

$$\sim \left(\begin{array}{cc|c} -b+1 & 0 & bI + bG + a + (I+G)\cdot(-b+1) \\ 0 & -b+1 & bI + bG + a \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cc|c} -b+1 & 0 & \cancel{bI} + \cancel{bG} + a - \cancel{bI} + I - \cancel{bG} + G \\ 0 & -b+1 & bI + bG + a \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cc|c} -b+1 & 0 & a + I + G \\ 0 & -b+1 & bI + bG + a \end{array}\right) \begin{array}{l} :(-b+1) \\ :(-b+1) \end{array} \sim$$

$$\sim \begin{array}{c} \begin{array}{cc} Y & C \end{array} \\ \left(\begin{array}{cc|c} 1 & 0 & \dfrac{a+I+G}{-b+1} \\ 0 & 1 & \dfrac{bI+bG+a}{-b+1} \end{array}\right) \end{array} \begin{array}{l} \mathrm{I} \\ \mathrm{II.} \end{array}$$

vyšla E ⟶ existuje řešení

II. $\boxed{C = \dfrac{bI + bG + a}{-b+1}}$

I. $\boxed{Y = \dfrac{a + I + G}{-b+1}}$