ROZBOR ZKOUŠKOVÝCH PŘÍKLADŮ 3

1. $\boxed{y = \dfrac{1+\ln x}{x}}$

1) $D_f = (0; \infty)$

$x \neq 0 \qquad x > 0 \qquad \longrightarrow x > 0$

2) $P_x \quad y = 0 \quad \longrightarrow \quad 0 = \dfrac{1+\ln x}{x}$

$$0 = 1 + \ln x$$
$$-1 = \ln x$$
$$e^{-1} = \cancel{e}^{\ln x}$$
$$e^{-1} = x$$

$P_x[e^{-1}; 0]$

$P_y \quad x \neq 0$

$\notin D_f$ neexistuje

$\ominus \qquad \oplus$

$0 \quad (e^{-2}) \quad e^{-1} \quad (e^{0}) \quad \infty$

$e^{-\infty} \qquad\qquad e^{\infty}$

$(e^{-2}): \dfrac{1+\cancel{\ln e}^{-2}}{e^{-2}} = \dfrac{1-2}{e^{-2}} = \dfrac{-}{+} = \ominus$

$(e^{0}): \dfrac{1+\cancel{\ln e}^{0}}{e^{0}} = \dfrac{1+0}{e^{0}} = \dfrac{+}{+} = \oplus$

3) $y' = \dfrac{(0+\frac{1}{x})\cdot x - (1+\ln x)\cdot 1}{(x)^2} = \dfrac{\frac{1}{\cancel{x}}\cdot\cancel{x} - 1 - \ln x}{x^2} = \dfrac{\cancel{1} - \cancel{1} - \ln x}{x^2} = \dfrac{-\ln x}{x^2}$

$y' = 0 \quad \longrightarrow \quad -\dfrac{\ln x}{x^2} = 0$

$$-\ln x = 0 \quad |(-1)$$
$$\ln x = 0$$
$$\cancel{e}^{\ln x} = e^{0}$$
$$x = e^{0} = 1$$

$\nearrow$ L.MAX $\searrow$

$0 \quad (e^{-1}) \quad 1 \quad (e^{1}) \quad \infty$

$e^{-\infty} \qquad e^{0} \qquad e^{\infty}$

$(e^{-1}): -\dfrac{\cancel{\ln e}^{-1}}{(e^{-1})^2} = \dfrac{-(-1)}{(e^{-1})^2} = \dfrac{+}{+} = \oplus$

$(e^{1}): -\dfrac{\cancel{\ln e}^{1}}{(e^{1})^2} = \dfrac{-1}{(e^{1})^2} = \dfrac{-}{+} = \ominus$

L.MAX $\quad x = 1$

$y = \dfrac{1+\ln 1}{1} = \dfrac{1+0}{1} = 1 \qquad [1; 1]$

4) $y'' = \dfrac{-\frac{1}{\cancel{x}}\cdot x^{\cancel{2}} - (-\ln x)\cdot 2x}{(x^2)^2} = \dfrac{-x + 2x\cdot \ln x}{x^4} = \dfrac{\cancel{x}[-1+2\ln x]}{x^{\cancel{4}\,3}} = \dfrac{-1+2\ln x}{x^3}$

$y'' = 0 \longrightarrow \dfrac{-1+2\ln x}{x^3} = 0$

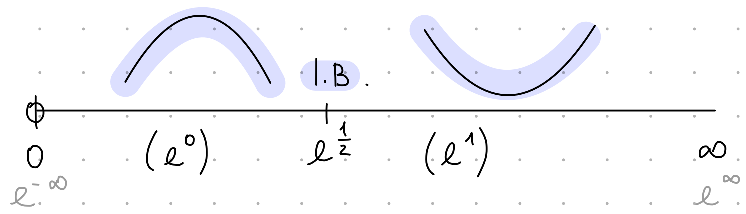
$-1+2\ln x = 0$

$2\ln x = 1$

$\ln x = \dfrac{1}{2}$

$\cancel{e}^{\cancel{\ln} x} = e^{\frac{1}{2}}$

$x = e^{\frac{1}{2}}$



(e^0): $\dfrac{-1+2\ln e^0}{(e^0)^3} = \dfrac{-1+2\cdot 0}{(e^0)^3} = \dfrac{-}{+} = -$

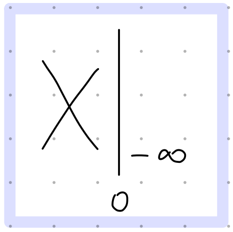
(e^1): $\dfrac{-1+2\ln e^1}{(e^1)^3} = \dfrac{-1+2\cdot 1}{(e^1)^3} = \dfrac{+}{+} = +$

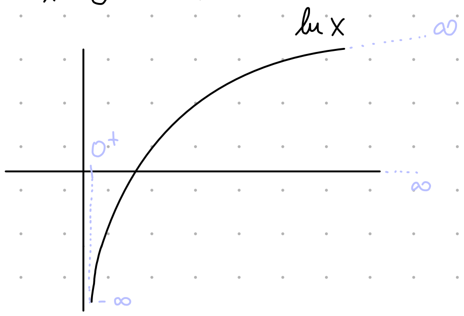
I.B. $x = e^{\frac{1}{2}}$

$y = \dfrac{1+\cancel{\ln e}^{\frac{1}{2}}}{e^{\frac{1}{2}}} = \dfrac{1+\frac{1}{2}}{e^{\frac{1}{2}}} = \dfrac{\frac{2+1}{2}}{e^{\frac{1}{2}}} = \dfrac{\frac{3}{2}}{e^{\frac{1}{2}}} = \dfrac{3}{2}e^{-\frac{1}{2}}$ $\quad [e^{\frac{1}{2}}; \frac{3}{2}e^{-\frac{1}{2}}]$

5) ABS

$\lim\limits_{x\to 0^+} \dfrac{1+\ln x}{x} = \dfrac{1+\ln 0^+}{0^+} = \dfrac{1-\infty}{0^+} = \dfrac{-\infty}{0^+} = -\infty$





6) ASS $\quad y = kx+q$

k: $\lim\limits_{x\to\infty} \dfrac{\frac{1+\ln x}{x}}{\underset{1}{x}} = \lim\limits_{x\to\infty} \dfrac{1+\ln x}{x^2} = \dfrac{1+\ln\infty}{\infty} = \dfrac{1+\infty}{\infty} = \left\|\dfrac{\infty}{\infty}\right\| \overset{L'H}{=}$

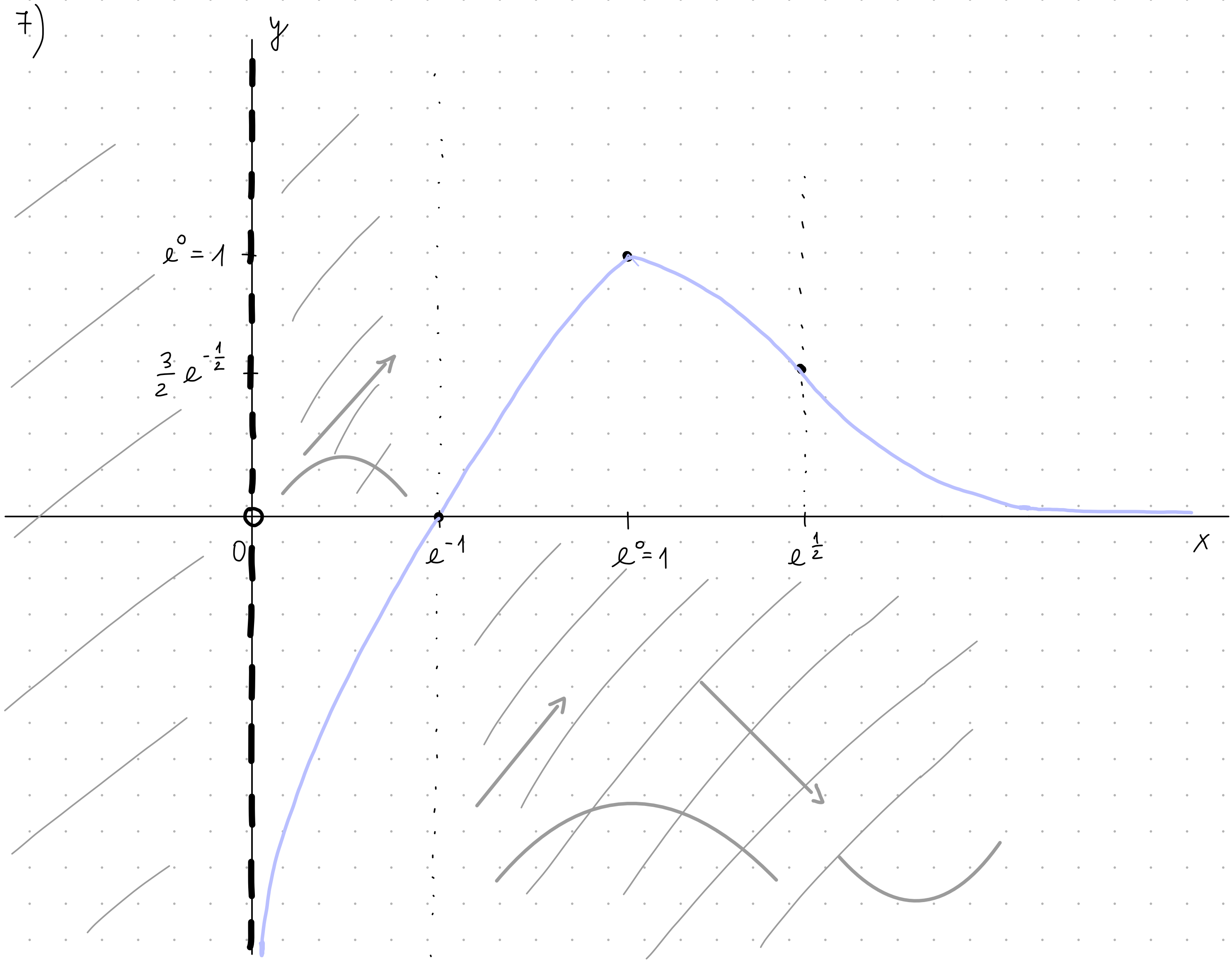
$\overset{L'H}{=} \lim\limits_{x\to\infty} \dfrac{0+\frac{1}{x}}{2x} = \lim\limits_{x\to\infty} \dfrac{1}{x}\cdot\dfrac{1}{2x} = \lim\limits_{x\to\infty} \dfrac{1}{2x^2} = \dfrac{1}{2\infty} = \dfrac{1}{\infty} = 0$

q: $\lim\limits_{x\to\infty} \dfrac{1+\ln x}{x} - \cancel{0x} = \dfrac{1+\ln\infty}{\infty} = \dfrac{1+\infty}{\infty} = \left\|\dfrac{\infty}{\infty}\right\| \overset{L'H}{=} \lim\limits_{x\to\infty} \dfrac{0+\frac{1}{x}}{1} =$

$= \lim\limits_{x\to\infty} \dfrac{1}{x} = \dfrac{1}{\infty} = 0$

$y = 0x + 0 = 0$ pro $x \to \infty$

7)



(2) $\boxed{\begin{aligned} Q &= U - P \\ Q &= V + \tfrac{2}{3}P \end{aligned}}$ $\longrightarrow$

$$\begin{aligned} -U &= -Q - P \quad |\,(-1) \\ -V &= -Q + \tfrac{2}{3}P \quad |\,(-1) \end{aligned}$$

$$\begin{aligned} U &= Q + P \\ V &= Q - \tfrac{2}{3}P \end{aligned} \longrightarrow \begin{array}{cccc} U & V & = & \text{zbytek} \\ 1U & +0V & = & Q + P \\ 0U & +1V & = & Q - \frac{2}{3}P \end{array}$$

$$\begin{array}{ccc} U & V & \text{čísla} \end{array}$$
$$\left(\begin{array}{cc|c} 1 & 0 & Q+P \\ 0 & 1 & Q - \frac{2}{3}P \end{array}\right)$$

$$|A| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$|A_1| = \begin{vmatrix} Q+P & 0 \\ Q - \frac{2}{3}P & 1 \end{vmatrix} = Q + P - 0 = Q + P \longrightarrow U = \frac{|A_1|}{|A|} = \frac{Q+P}{1} = Q + P$$

$$|A_2| = \begin{vmatrix} 1 & Q+P \\ 0 & Q - \frac{2}{3}P \end{vmatrix} = Q - \frac{2}{3}P - 0 = Q - \frac{2}{3}P \longrightarrow V = \frac{|A_2|}{|A|} = \frac{Q - \frac{2}{3}P}{1} = Q - \frac{2}{3}P$$

③ $\boxed{y = \operatorname{arctg} \frac{2x+1}{\sqrt{3}} \qquad T[-\tfrac{1}{2}; ?]}$

$$y'(-\tfrac{1}{2}) = \frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2} \cdot \frac{2\cdot\sqrt{3} - \cancel{(2x+1)\cdot 0}}{(\sqrt{3})^2} \Rightarrow \frac{1}{1+\left(\frac{2\cdot(-\frac{1}{2})+1}{\sqrt{3}}\right)^2} \cdot \frac{2\cancel{\sqrt{3}}}{3} =$$

$3 \rightarrow \sqrt{3}\,\cancel{\sqrt{3}}$

$$= \frac{1}{1+\left(\frac{\cancel{-1+1}}{\sqrt{3}}\right)^2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{1} \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

④

$$\boxed{\left(\begin{array}{ccccc|c} 2 & -3 & 0 & 1 & 3 & 3 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 3 & 3 & 0 & -3 & 4 \\ 0 & 0 & 0 & 1 & 0 & 4 \end{array}\right)} \sim \left(\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 0 & 1 \\ 2 & -3 & 0 & 1 & 3 & 3 \\ 1 & 3 & 3 & 0 & -3 & 4 \\ 0 & 0 & 0 & 1 & 0 & 4 \end{array}\right) \begin{array}{c} (-2)\ (-1) \end{array} \sim$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & -3 & -2 & 1 & 3 & 1 \\ 0 & 3 & 2 & 0 & -3 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \end{array}\right) \sim \begin{array}{cccccc|cc} x_1 & x_2 & x_3 & x_4 & x_5 & & & \\ 1 & 0 & 1 & 0 & 0 & & 1 & \mathrm{I.} \\ 0 & -3 & -2 & 1 & 3 & & 1 & \mathrm{II.} \\ 0 & 0 & 0 & 1 & 0 & & 4 & \mathrm{III.} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} & & \cancel{4} & \end{array}$$

<u>rozbor řešení</u>

$h(A) = 3$

$h(A_r) = 3$

$\longrightarrow$ existuje řešení

$n = 5$

$p = n - h(A) = 5 - 3 = 2$

$\longrightarrow$ 2 parametry, ∞ řešení

III. $x_4 = 4$

II. $-3x_2 - 2x_3 + x_4 + 3x_5 = 1$

$-3x_2 - 2x_3 + 4 + 3x_5 = 1$

$-3x_2 - 2x_3 + 3x_5 = -3$

I. $x_1 + x_3 = 1$

$x_1 + q = 1$

$x_1 = 1 - q$

$x_5 = p \ ; p \in \mathbb{R}$

$x_3 = q \ ; q \in \mathbb{R}$

$-3x_2 - 2q + 3p = -3$

$-3x_2 = -3 + 2q - 3p \quad |:(-3)$

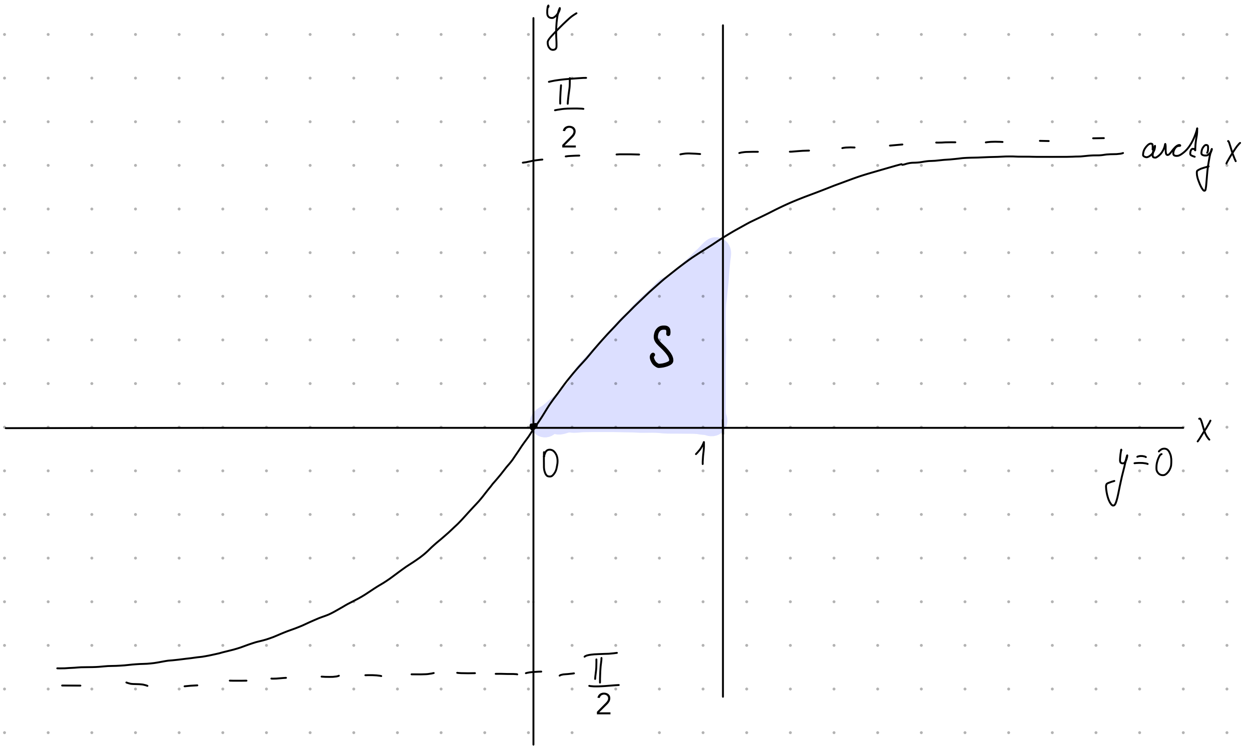
$x_2 = 1 - \frac{2}{3}q + p$

OŘ: $\vec{x} = (1-q;\ 1-\frac{2}{3}q + p;\ q;\ 4;\ p)$ $\quad p; q \in \mathbb{R}$

ZŘ: $\vec{x} = (1; 1; 0; 4; 0)$ $\quad p = q = 0$

PŘ: $\vec{x} = (1; 2; 0; 4; 1)$ $\quad q = 0;\ p = 1$

⑤ $\boxed{y = \operatorname{arctg} x \quad y = 0 \quad x = 1}$



$$S = \int_0^1 \operatorname{arctg} x - 0 \, dx = \int_0^1 1 \cdot \operatorname{arctg} x \, dx = \left| \begin{matrix} u = \operatorname{arctg} x & u' = \frac{1}{1+x^2} \\ v' = 1 & v = x \end{matrix} \right| =$$

$$= \Big[x \cdot \operatorname{arctg} x \Big]_0^1 - \frac{1}{2} \int_0^1 \frac{2x}{\underbrace{1+x^2}_{\to 2x}} dx = \Big[x \cdot \operatorname{arctg} x \Big]_0^1 - \Big[\frac{1}{2} \ln |1+x^2| \Big]_0^1 =$$

$$= \Big[x \cdot \operatorname{arctg} x - \frac{1}{2} \ln |1+x^2| \Big]_0^1 =$$

$$= 1 \cdot \operatorname{arctg} 1 - \frac{1}{2} \ln |1+1^2| - \left(\cancel{0 \cdot \operatorname{arctg} 0} - \frac{1}{2} \ln |1+0^2| \right) =$$

$$= \frac{\pi}{4} - \frac{1}{2} \ln 2 + \cancel{\frac{1}{2} \underbrace{\ln 1}_{0}} = \boxed{\frac{\pi}{4} - \frac{1}{2} \ln 2}$$

⑥ $\boxed{z = \arcsin(x-3) + \dfrac{\sqrt{xy^2}}{x+1}}$

$-1 \le x-3 \le 1$

A ZÁROVEŇ

$-1 \le x-3$

$2 \le x$

$x \ge 2$

$x-3 \le 1$

$x \le 4$

$x+1 \ne 0$

$x \ne -1$

$xy^2 \ge 0$

NEBO

$+ \cdot +$

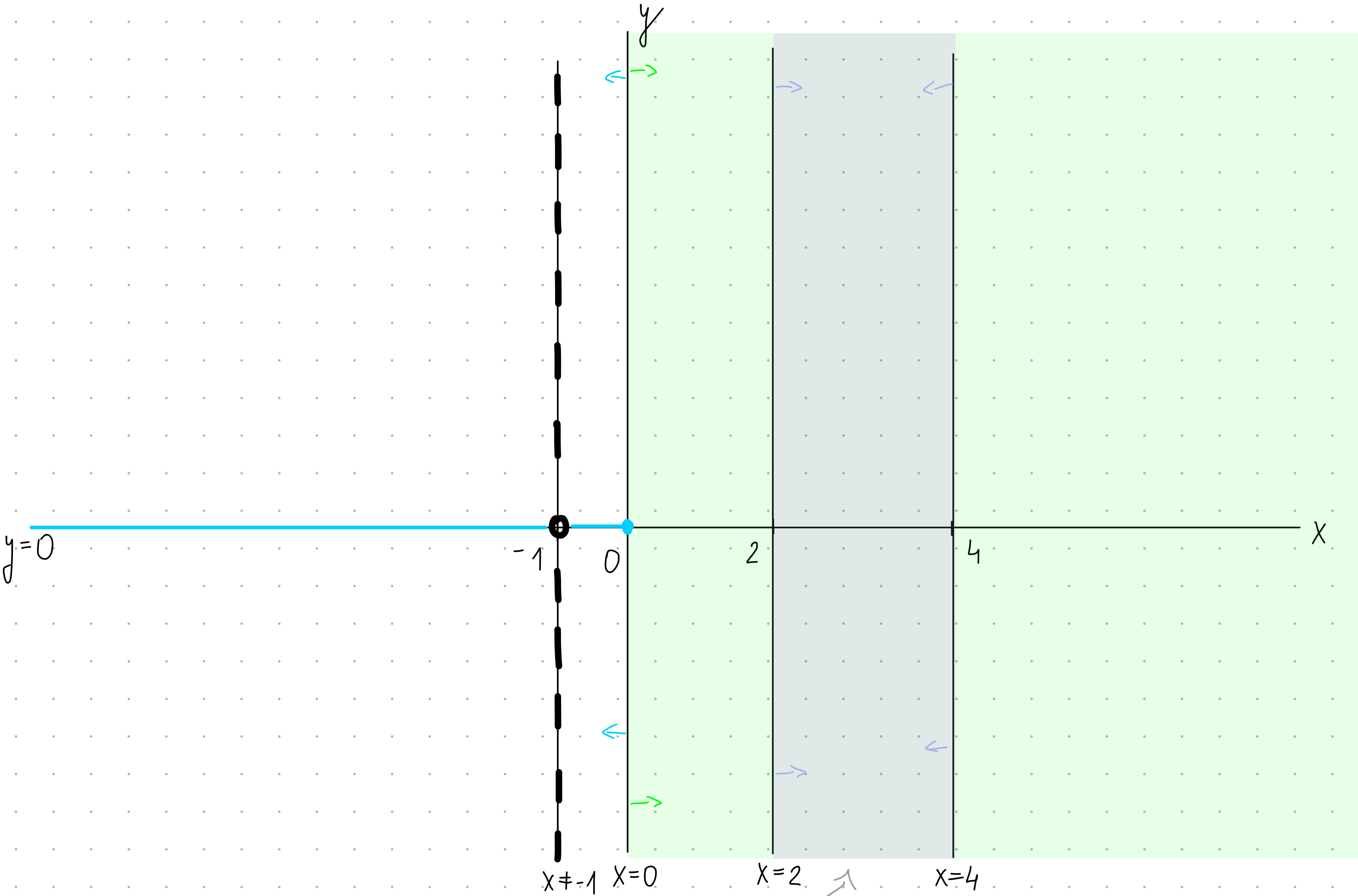
$x \ge 0 \wedge y^2 \ge 0$

vždy

$- \cdot -$

$x \le 0 \wedge y^2 \le 0$

$y = 0$



$D_f =$

neomezená → ani otevřená ani uzavřená } NENÍ OBLAST

souvislá

(7)

$$\boxed{z = e^{-x}\left(4x^2 + y^2 - 3xy + 5y + 2\right) \qquad g: -y + 2x = 5}$$

$\rightarrow -y = 5 - 2x \quad |\cdot(-1)$

$y = -5 + 2x$

1) $D_f = \mathbb{R} \times \mathbb{R}$

2) $z = e^{-x}\left(4x^2 + (-5+2x)^2 - 3x(-5+2x) + 5(-5+2x) + 2\right)$

$z = e^{-x}\left(4x^2 + 25 - 20x + 4x^2 + 15x - 6x^2 - 25 + 10x + 2\right)$

$z = e^{-x} \cdot \left(2x^2 + 5x + 2\right)$

3) $z'_x = e^{-x} \cdot (-1) \cdot (2x^2 + 5x + 2) + e^{-x}(4x + 5) = e^{-x}\left[-2x^2 - 5x - 2 + 4x + 5\right] =$

$= e^{-x}\left(-2x^2 - x + 3\right)$

$+$

$z'_x = 0 \quad \longrightarrow \quad e^{-x} \cdot (-2x^2 - x + 3) = 0$

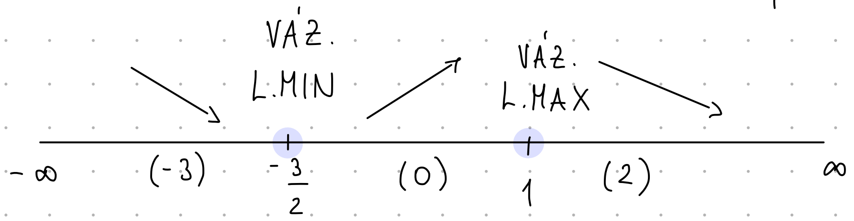
$e^{-x} \neq 0 \qquad -2x^2 - x + 3 = 0$

$$x_{1,2} = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot (-2) \cdot 3}}{2 \cdot (-2)} = \frac{1 \pm \sqrt{1 + 24}}{-4}$$

$$x_{1,2} = \frac{1 \pm 5}{-4}$$

$x_1 = \frac{6}{-4} = -\frac{3}{2}$

$x_2 = \frac{-4}{-4} = 1$



$(-3): \; -2 \cdot (-3)^2 - (-3) + 3 = -2 \cdot 9 + 3 + 3 = -18 + 6 = \ominus$

$(0): \; -2 \cdot 0^2 - 0 + 3 = 0 - 0 + 3 = \oplus$

$(2): \; -2 \cdot 2^2 - 2 + 3 = -2 \cdot 4 - 2 + 3 = -8 + 1 = \ominus$

4) VÁZ. L. MIN $\quad x = -\frac{3}{2}$

$y = -5 + \cancel{2}\left(-\frac{3}{\cancel{2}}\right) = -5 - 3 = -8$

$z = e^{+\frac{3}{2}}\left(4 \cdot \left(-\frac{3}{2}\right)^2 + (-8)^2 - 3 \cdot \left(-\frac{3}{2}\right) \cdot (-8) + 5 \cdot (-8) + 2\right) = e^{+\frac{3}{2}}(-1) = -e^{+\frac{3}{2}}$

VÁZ. L. MAX $\quad x = 1$

$y = -5 + 2 \cdot 1 = -5 + 2 = -3$

$z = e^{-1} \cdot \left(4 \cdot 1^2 + (-3)^2 - 3 \cdot 1 \cdot (-3) + 5 \cdot (-3) + 2\right) = e^{-1} \cdot (9) = 9e^{-1}$

5) funkce má l. váz. min $\left[-\frac{3}{2}; -8\right]$ v bodě $-e^{+\frac{3}{2}}$

l. váz. max $[1; -3]$ v bodě $9e^{-1}$