ROZBOR ZKOUŠKOVÝCH PŘÍKLADŮ 4

① $\boxed{z = 3x^2 - y + 2x \qquad g: y - x + 1 = 0}$

1) $D_f = \mathbb{R} \times \mathbb{R}$

$\lambda y - \lambda x - \lambda$

2) $L = 3x^2 - y + 2x + \lambda \cdot (y - x + 1)$

3) $L'_x = 6x + 2 - \lambda$

$L'_y = -1 + \lambda$

$L'_\lambda = g = y - x + 1$

4)
$$6x + 2 - \lambda = 0$$
$$6x + 2 - 1 = 0$$
$$6x = -1$$
$$x = -\frac{1}{6}$$

$$-1 + \lambda = 0$$
$$\lambda = 1$$

$$y - x + 1 = 0$$
$$y - \left(-\frac{1}{6}\right) + 1 = 0$$
$$y + \frac{1}{6} + 1 = 0$$
$$y + \frac{1+6}{6} = 0$$
$$y = -\frac{7}{6}$$

$P\left[-\frac{1}{6}; -\frac{7}{6}\right] \quad \lambda = 1$

5) $L''_{xx} = 6$

$L''_{yy} = 0$

$L''_{xy} = 0$

$g'_x = -1$

$g'_y = 1$

6) ✓

7)
$$D = -\begin{vmatrix} 6 & 0 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix} = -[0 - 6] = 6 > 0$$

$0 + 6 + 0 = 6 \qquad 0 + 0 + 0 = 0$

L.VÁZ.MIN

8) $z = 3 \cdot \left(-\frac{1}{6}\right)^2 - \left(-\frac{7}{6}\right) + 2 \cdot \left(-\frac{1}{6}\right) = \cancel{3} \cdot \frac{1}{\cancel{36}_{12}} + \frac{7}{6} - \frac{2}{6} = \frac{1}{12} + \frac{5}{6} = \frac{1+10}{12} = \frac{11}{12}$

L.VÁZ.MIN $\left[-\frac{1}{6}; -\frac{7}{6}\right]$ v bodě $z = \frac{11}{12}$

$\lambda = 1$

②

$$\boxed{z = \sqrt{6 - x - x^2} - \ln(x^2 + y^2 - 4)}$$

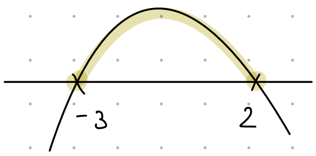
$6 - x - x^2 \geq 0$

$6 - x - x^2 = 0$
$-x^2 - x + 6 = 0$

$x_{1,2} = \dfrac{+1 \pm \sqrt{(-1)^2 - 4\cdot(-1)\cdot 6}}{2\cdot(-1)} = \dfrac{1 \pm \sqrt{1+24}}{-2} = \dfrac{1 \pm 5}{-2}$

$x_1 = \dfrac{6}{-2} = -3$

$x_2 = \dfrac{-4}{-2} = 2$



na x mezi -3 a 2

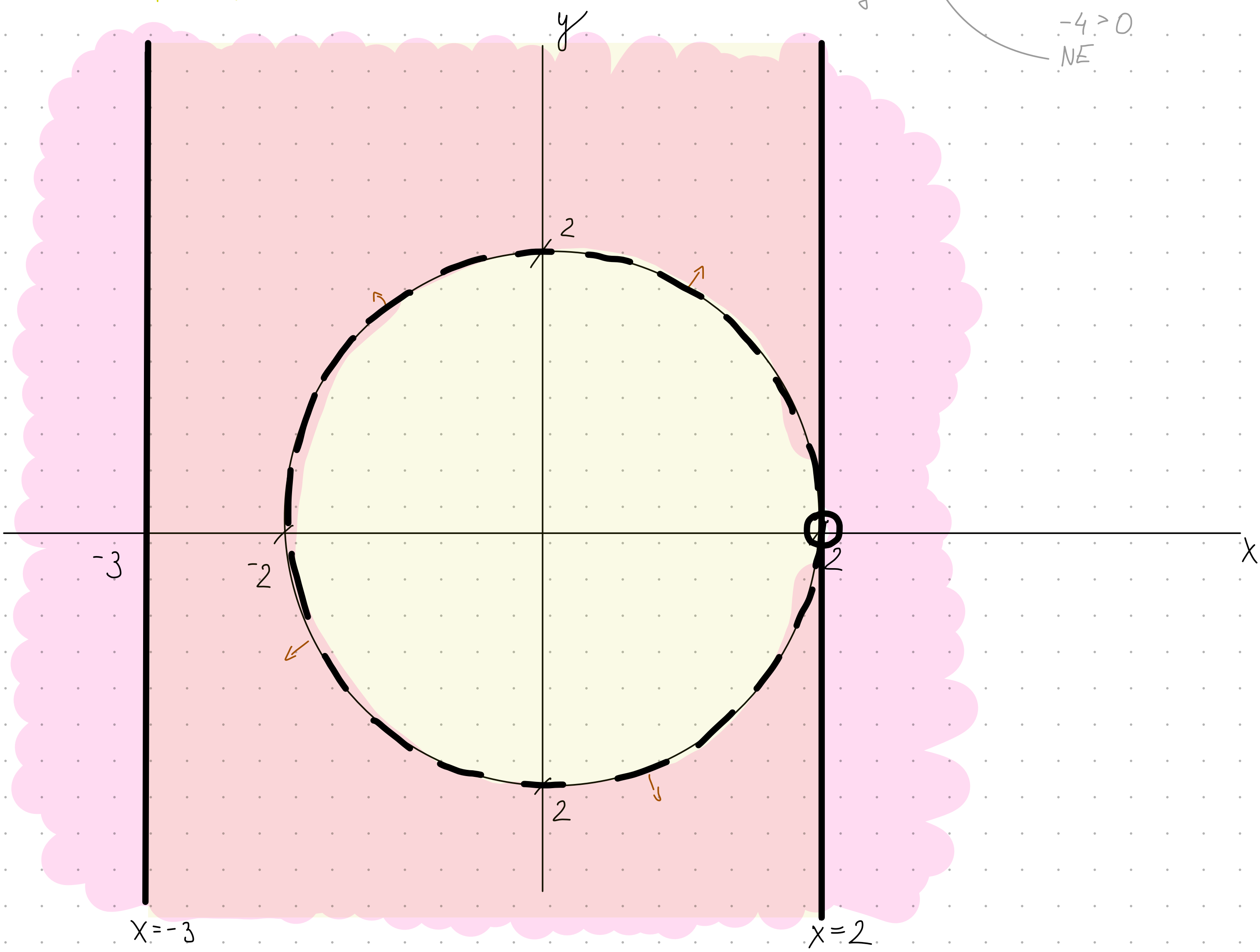
$x^2 + y^2 - 4 > 0$

$x^2 + y^2 > 4$ $\quad k: S[0;0]; r = \sqrt{4} = 2$

dosadím [0;0]
UVNITŘ

$0^2 + 0^2 - 4 > 0$
$0 - 4 > 0$
$-4 > 0$
NE

Df není



Df =

neomezený
není otevřený ani uzavřený
souvislý

} NENÍ OBLAST

③ $\boxed{\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 2 & 3 & 3 & 3 & -1 \\ 1 & 5 & 2 & 3 & -2 \\ 5 & -9 & 5 & 1 & 4 \end{array}\right)}$ $(-2)\ (-1)\ (-5)$ $\sim$

báze $\{x_1; x_2; x_4\}$

$$\sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & 7 & 1 & 3 & -3 \\ \cancel{0} & \cancel{7} & \cancel{1} & \cancel{3} & \cancel{-3} \\ 0 & 1 & 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 7 & 1 & 3 & -3 \end{array}\right) (-7) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & -4 & 4 \end{array}\right) (4) \sim \left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 & 4 \end{array}\right) (2) \sim$$

$$\sim \left(\begin{array}{cccc|c} 2 & 0 & 3 & 0 & 2 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 & 4 \end{array}\right) \begin{array}{l} :2 \\ :4 \\ :(-4) \end{array} \sim \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & & \\ \multicolumn{5}{c}{\left(\begin{array}{cccc|c} 1 & 0 & \frac{3}{2} & 0 & 1 \\ 0 & 1 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & -\frac{1}{4} & 1 & -1 \end{array}\right)} & \begin{array}{l} \mathrm{I}. \\ \mathrm{II}. \\ \mathrm{III}. \end{array} \end{array}$$

$\left.\begin{array}{l} h(A) = 3 \\ h(A_r) = 3 \end{array}\right\}$ existuje řešení $\qquad \left.\begin{array}{l} n = 4 \\ h(A) = 3 \end{array}\right\}$ $p = 4 - 3 = 1$ $\quad$ ∞ řešení
x_3

$\mathrm{III}.\ -\frac{1}{4}x_3 + x_4 = -1$
$x_4 = -1 + \frac{1}{4}x_3$

$\mathrm{II}.\ x_2 + \frac{1}{4}x_3 = 0$
$x_2 = -\frac{1}{4}x_3$

$\mathrm{I}.\ x_1 + \frac{3}{2}x_3 = 1$
$x_1 = 1 - \frac{3}{2}x_3$

OŘ: $\vec{x} = \{1 - \frac{3}{2}x_3; -\frac{1}{4}x_3; x_3; -1 + \frac{1}{4}x_3\}$ $\qquad x_3 \in \mathbb{R}$

ZŘ: $\vec{x} = \{1; 0; 0; -1\}$ $\qquad x_3 = 0$

PŘ: $\vec{x} = \{1 - \frac{3}{\cancel{2}} \cdot \cancel{4}^{2}; -\frac{1}{\cancel{4}} \cdot \cancel{4}; 4; -1 + \frac{1}{\cancel{4}} \cdot \cancel{4}\}$ $\qquad x_3 = 4$
$= \{-5; -1; 4; 0\}$

(4) $\boxed{y^2 y' - \frac{\ln x}{x}\sqrt{y} = 0}$

$$y^2 y' = \frac{\ln x}{x}\sqrt{y} \quad \Big| \cdot y^2$$

$$y' = \frac{\ln x}{x} \cdot \frac{\sqrt{y}}{y^2}$$

$$\frac{dy}{dx} = \frac{\ln x}{x} \cdot \frac{\sqrt{y}}{y^2} \quad \Big| \cdot dx$$

$$dy = \frac{\ln x}{x} \cdot \frac{\sqrt{y}}{y^2} \cdot dx \quad \Big| \cdot \frac{y^2}{\sqrt{y}}$$

$$\int \frac{y^2}{\sqrt{y}}\, dy = \int \frac{\ln x}{x}\, dx$$

$$\frac{2}{5}\left(\sqrt{y}\right)^5 = \frac{\ln^2 x}{2} + C \quad \Big| \cdot \frac{5}{2}$$

$$\left(\sqrt{y}\right)^5 = \frac{5}{2} \cdot \frac{\ln^2 x}{2} + \frac{5}{2} \cdot C \xrightarrow{} C \quad \Big| \sqrt[5]{\ }$$

$$\sqrt{y} = \sqrt[5]{\frac{5}{4}\ln^2 x + C} \quad \Big|^2$$

$$\boxed{y = \sqrt[5]{\left(\frac{5}{4}\ln^2 x + C\right)^2}} \quad \text{OŘ.}$$

S.Ř. $y = 0$

$$\sqrt{y} \neq 0 \longrightarrow y \neq 0$$

$$\cancel{y \geq 0}$$

$$\int \frac{y^2}{\sqrt{y}}\, dy = \left| \begin{array}{l} \sqrt{y} = s \quad \big|^2 \\ y = s^2 \\ dy = 2s\, ds \end{array} \right| = \int \frac{s^4}{\cancel{s}}\, 2\cancel{s}\, ds =$$

$$= 2\int s^4\, ds = 2 \cdot \frac{s^5}{5} = \frac{2}{5}\left(\sqrt{y}\right)^5$$

$$\int \frac{\ln x}{x}\, dx = \left| \begin{array}{l} \ln x = s \\ \frac{1}{x}dx = ds \end{array} \right| = \int s\, ds =$$

$$= \frac{s^2}{2} = \frac{(\ln x)^2}{2} = \frac{\ln^2 x}{2}$$

5 $\boxed{\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+2x+2}\,dx} = \lim_{x\to\infty} \int \frac{1}{x^2+2x+2}\,dx - \lim_{x\to-\infty} \int \frac{1}{x^2+2x+2}\,dx$

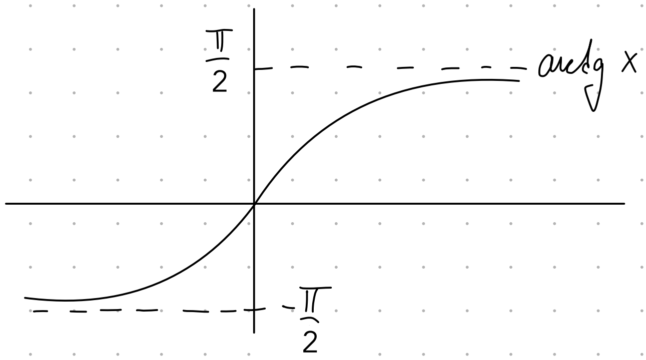
$$\longrightarrow \int \frac{1}{x^2+2x+2}\,dx = \int \frac{1}{(x+1)^2+1}\,dx = \frac{1}{1}\left(\operatorname{arctg} \frac{x+1}{1}\right)\cdot\frac{1}{1} =$$

$$\hookrightarrow x^2+2x+2 = (x+1)^2-(1)^2+2 = (x+1)^2-1+2 = (x+1)^2+1$$

$$= \operatorname{arctg}(x+1)$$

$$\longrightarrow \lim_{x\to\infty} \operatorname{arctg}(x+1) - \lim_{x\to-\infty} \operatorname{arctg}(x+1) = \operatorname{arctg}(\infty+1) - \operatorname{arctg}(-\infty+1) =$$

$$= \operatorname{arctg}\infty - \operatorname{arctg}(-\infty) = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}+\frac{\pi}{2} = \frac{\cancel{2}\pi}{\cancel{2}} = \boxed{\pi}$$ konverguje



(6)

$$p(x)=\begin{cases} 0{,}7 & \text{pro } x=0 \\ 0{,}2 & \text{pro } x=1 \\ 0{,}1 & \text{pro } x=2 \\ 0 & \text{jinak} \end{cases}$$

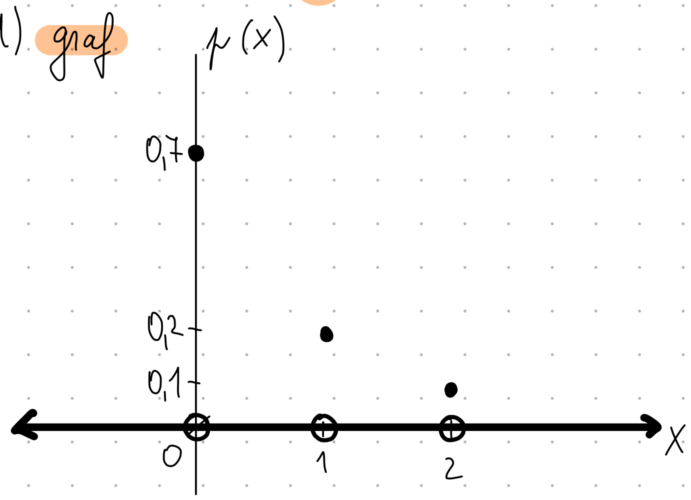
a) $E(x) = \cancel{0\cdot 0{,}7} + 1\cdot 0{,}2 + 2\cdot 0{,}1 = 1\cdot\frac{2}{10} + 2\cdot\frac{1}{10} = \frac{2}{10}+\frac{2}{10} = \frac{4}{10} = \frac{2}{5}$

b) $D(x) = \cancel{0^2\cdot 0{,}7} + 1^2\cdot 0{,}2 + 2^2\cdot 0{,}1 - \left(\frac{2}{5}\right)^2 = 1\cdot\frac{2}{10} + 4\cdot\frac{1}{10} - \frac{4}{25} =$

$= \frac{2}{10} + \frac{4}{10} - \frac{4}{25} = \frac{20+40-16}{100} = \frac{44}{100} = \frac{11}{25}$

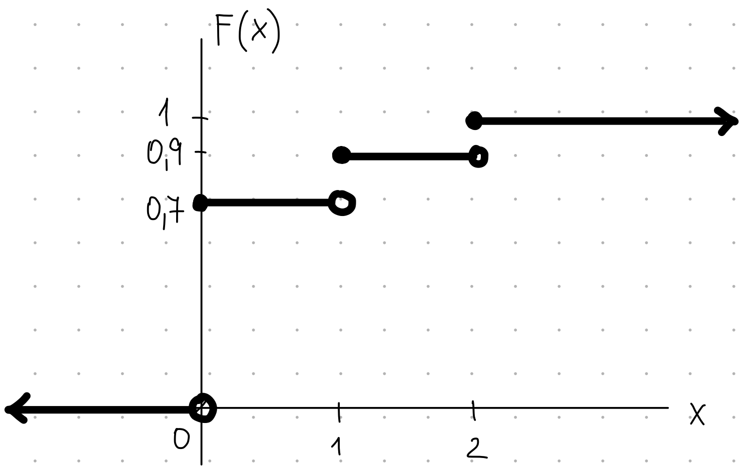
c) $\sqrt{D(x)} = \sqrt{\frac{11}{25}} = \frac{\sqrt{11}}{5}$

d) graf



e) distribuční fce $F(x) = P(X\leq x)$

$F(0) = P(X\leq 0) = P(0) = 0{,}7$
$F(1) = P(X\leq 1) = P(1)+P(0) = 0{,}2+0{,}7 = 0{,}9$
$F(2) = P(X\leq 2) = P(2)+P(1)+P(0) = 0{,}1+0{,}2+0{,}7 = 1$



$$F(x)=\begin{cases} 0 & \text{pro } x\in(-\infty;0) \\ 0{,}7 & \text{pro } x\in[0;1) \\ 0{,}9 & \text{pro } x\in[1;2) \\ 1 & \text{pro } x\in[2;\infty) \end{cases}$$

(7)

$$F(x)=\begin{cases}0 & \text{po } x\le 1\\ x\cdot\ln x - x+1 & \text{po } x\in(1;e)\\ 1 & \text{po } x\ge e\end{cases}$$

$F(x)=x\cdot\ln x - x+1 \quad \text{po } x\in(1;e)$

a) $f(x)=[F(x)]'=(x\cdot\ln x - x+1)'=1\cdot\ln x+\cancel{x}\cdot\frac{1}{\cancel{x}}-1+0=$

$=\ln x+\cancel{1}-\cancel{1}=\ln x$

b) $E(x)=\int_1^e x\cdot\ln x\,dx$

$$\int x\cdot\ln x\,dx=\left|\begin{matrix}u=\ln x & u'=\frac{1}{x}\\ v'=x & v=\frac{x^2}{2}\end{matrix}\right|=\frac{x^2}{2}\cdot\ln x-\int\frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}}}{2}\,dx=$$

$$=\frac{x^2}{2}\cdot\ln x-\frac{1}{2}\int x\,dx=\frac{x^2}{2}\cdot\ln x-\frac{1}{2}\cdot\frac{x^2}{2}=\frac{x^2}{2}\cdot\ln x-\frac{x^2}{4}$$

$$E(x)=\left[\frac{x^2}{2}\cdot\ln x-\frac{x^2}{4}\right]_1^e=\frac{e^2}{2}\cdot\cancel{\ln e}^{\,1}-\frac{e^2}{4}-\left(\frac{1^2}{2}\cdot\underbrace{\ln 1}_{0}-\frac{1^2}{4}\right)=$$

$$=\frac{e^2}{2}\cdot 1-\frac{e^2}{4}-\cancel{\frac{1}{2}\cdot 0}+\frac{1}{4}=\frac{e^2}{2}-\frac{e^2}{4}+\frac{1}{4}=\frac{2e^2-e^2+1}{4}=\frac{e^2+1}{4}$$

c) $P(X>\sqrt{e})=P(\sqrt{e}<X)=P(\sqrt{e}<X<e)=F(e)-F(\sqrt{e})=$

$=e\cdot\cancel{\ln e}^{\,1}-e+1-(\sqrt{e}\cdot\ln\sqrt{e}-\sqrt{e}+1)=$

$=e\cdot 1-e+1-\sqrt{e}\cdot\ln e^{\frac{1}{2}}+\sqrt{e}-1=\cancel{e}-\cancel{e}+\cancel{1}-\sqrt{e}\cdot\frac{1}{2}+\sqrt{e}-\cancel{1}=$

$=-\frac{\sqrt{e}}{2}+\sqrt{e}=\frac{-\sqrt{e}+2\sqrt{e}}{2}=\frac{\sqrt{e}}{2}$